PATHFINDER®
The No.1 Institute in Eastern India

WBJEE (Engineering) 2014

Solution

PHYSICS

Category : I

1. A small metal sphere of radius a is falling with a velocity v through a vertical column of a viscous liquid. If the coefficient of viscosity of the liquid is η, then the sphere encounters an opposing force of

(A) $6\pi\eta a^2 v$ (B) $\frac{6\eta v}{\pi a}$ (C) $6\pi\eta a v$ (D) $\frac{\pi\eta v}{6a^3}$

Solution : (C)

2. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(A) $\frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$ (B) $\frac{h_1t_1^2 + h_2t_2^2}{h_2t_1 + h_1t_2}$ (C) $\frac{h_1t_2^2 + h_2t_1^2}{h_1t_2 + h_2t_1}$ (D) $\frac{h_1t_1^2 - h_2t_2^2}{h_1t_1 - h_2t_2}$

Solution : (A)

3. In which of the following pairs, the two physical quantities have different dimensions ?

(A) Planck's constant and angular momentum (B) Impulse and linear momentum

(C) Moment of inertia and moment of a force (D) Energy and torque

Solution : (C)

4. A very small circular loop of radius a is initially (at t = 0) coplanar and concentric with a much larger fixed circular loop of radius b. A constant curent I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is

(A) $\frac{\pi a^2\mu_0 I}{2b}\omega\cos(\omega t)$ (B) $\frac{\pi a^2\mu_0 I}{2b}\omega\sin(\omega^2 t^2)$ (C) $\frac{\pi a^2\mu_0 I}{2b}\omega\sin(\omega t)$ (D) $\frac{\pi a^2\mu_0 I}{2b}\omega\sin^2(\omega t)$

Solution : (C)

5. An electron in a circular orbit of radius .05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in Am^2)

(A) 2.16×10^{-23} (B) 3.21×10^{-22} (C) 3.21×10^{-24} (D) 1.26×10^{-23}

Solution : (D)

6. A drop of some liquid of volume 0.04 cm^3 is placed on the surface of a glass slide. Then another glass slide is placed on it in such a way that the liquid forms a thin layer of area 20 cm^2 between the surfaces of the two slides. To separate the slides a force of 16×10^5 dyne has to be applied normal to the surfaces. The surface tension of the liquid is (in dyne-cm^{-1})

(A) 60 (B) 70 (C) 80 (D) 90

Solution : (C)

7. In a transistor output characteristics commonly used in common emitter configuration, the base current I_B, the collector current I_C and the collector-emitter voltage V_{CE} have values of the following orders of magnitude in the active region

(A) I_B and I_C both are in μA and V_{CE} in Volts
(B) I_B is in μA and I_C is in mA and V_{CE} in Volts
(C) I_B is in mA and I_C is in μA and V_{CE} in mV
(D) I_B is in mA and I_C is in mA and V_{CE} in mV

Solution : (B)

8. If n denotes a positive integer, h the Planck's constant, q the charge and B the magnetic field, then the quantity $\left(\frac{nh}{2\pi qB}\right)$ has the dimension of

(A) area (B) length (C) speed (D) acceleration

Solution : (A)

9. In the circuit shown assume the diode to be ideal. When V_i increases from 2 V to 6 V, the change in the current is (in mA)

(A) zero (B) 20 (C) $\frac{80}{3}$ (D) 40

Solution : (B)

10. A galvanometer having internal resistance 10 Ω requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full-scale deflection at 120 V, we need to connect a resistance of

(A) 11990 Ω in series (B) 11990 Ω in parallel (C) 12010 Ω in series (D) 12020 Ω in parallel

Solution : (A)

11. Three capacitors, 3 μF, 6 μF and 6 μF are connected in series to a source of 120 V. The potential difference, in volts, across the 3 μF capacitor will be

(A) 24 (B) 30 (C) 40 (D) 60

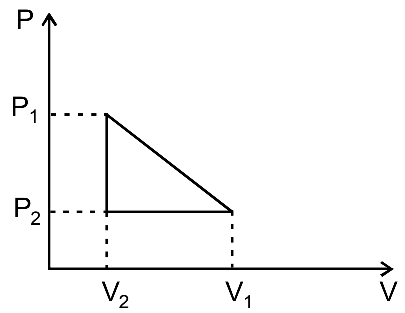
Solution : (D)

12. Consider three vectors $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}, \vec{B} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{C} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. A vector $\vec{X}$ of the form $\alpha\vec{A} + \beta\vec{B}$ (α and β are numbers) is perpendicular to $\vec{C}$. The ratio of α and β is

(A) 1 : 1 (B) 2 : 1 (C) – 1 : 1 (D) 3 : 1

Solution : (A)

13. One mole of a van der Waal's gas obeying the equation $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ undergoes the quasi-static cyclic process which is shown in the P-V diagram. The net heat absorbed by the gas in this process is



(A) $\frac{1}{2}(P_1-P_2)(V_1-V_2)$ (B) $\frac{1}{2}(P_1+P_2)(V_1-V_2)$

(C) $\frac{1}{2}\left(P_1+\frac{a}{V_1^2}-P_2-\frac{a}{V_2^2}\right)(V_1-V_2)$ (D) $\frac{1}{2}\left(P_1+\frac{a}{V_1^2}+P_2+\frac{a}{V_2^2}\right)(V_1-V_2)$

Solution : (A)

14. A scientist proposes a new temperature scale in which the ice point is 25 X (X is the new unit of temperature) and the steam point is 305 X. The specific heat capacity of water in this new scale is (in $JKg^{-1}X^{-1}$)
(A) 4.2×10^3 (B) 3.0×10^3 (C) 1.2×10^3 (D) 1.5×10^3

Solution : (D)

15. The intensity of magnetization of a bar magnet is $5.0\times10^4\ Am^{-1}$. The magnetic length and the area of cross section of the magnet are 12 cm and 1 cm^2 respectively. The magnitude of magnetic moment of this bar magnet is (in SI unit)
(A) 0.6 (B) 1.3 (C) 1.24 (D) 2.4

Solution : (A)

16. An infinite sheet carrying a uniform surface charge density σ lies on the xy-plane. The work done to carry a charge q from the point $\vec{A}=a\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ to the point $\vec{B}=a\left(\hat{i}-2\hat{j}+6\hat{k}\right)$ (where a is a constant with the dimension of length and ε_0 is the permittivity of free space) is

(A) $\frac{3\sigma aq}{2\varepsilon_0}$ (B) $\frac{2\sigma aq}{\varepsilon_0}$ (C) $\frac{5\sigma aq}{2\varepsilon_0}$ (D) $\frac{3\sigma aq}{\varepsilon_0}$

Solution : (A)

17. A luminous object is separated from a screen by distance d. A convex lens is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is

(A) 4d (B) 2d (C) d/2 (D) d/4

Solution : (D)

18. Four cells, each of emf E and internal resistance r, are connected in series across an external resistance R. By mistake one of the cells is connected in reverse. Then the current in the external circit is

(A) $\frac{2E}{4r+R}$ (B) $\frac{3E}{4r+R}$ (C) $\frac{3E}{3r+R}$ (D) $\frac{2E}{3r+R}$

Solution : (A)

19. Consider two concentric spherical metal shells of radii r_1 and $r_2 (r_2 > r_1)$. If the outer shell has a charge q and the inner one is grounded, the charge on the inner shell is

(A) $\frac{-r_2}{r_1}q$ (B) zero (C) $\frac{-r_1}{r_2}q$ (D) $-q$

Solution : (C)

20. Consider a blackbody radiation in a cubical box at absolute temperature T. If the length of each side of the box is doubled and the temperature of the walls of the box and that of the radiation is halved, then the total energy

(A) halves (B) doubles (C) quadruples (D) remains the same

Solution : (C)

21. The displacement of a particle in a periodic moment is given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$. This displacement may be considered as the result of superposition of n independent harmonic oscillations. Here n is

(A) 1 (B) 2 (C) 3 (D) 4

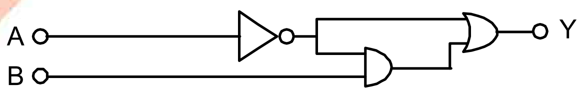
Solution : (C)

22. One mole of an ideal monoatomic gtas is heated at a constant pressure from 0°C to 100°C. Then the change in the internal energy of the gas is (Given $R = 8.32\ Jmol^{-1}K^{-1}$)

(A) 0.83×10^3 J (B) 4.6×10^3 J (C) 2.08×10^3 J (D) 1.25×10^3 J

Solution : (D)

23. The output Y of the logic circuit given below is



(A) $\bar{A}+B$ (B) $\bar{A}$ (C) $\overline{(\bar{A}+B)}.\bar{A}$ (D) $\overline{(\bar{A}+B)}.A$

Solution : (B)

24. A whistle whose air column is open at both ends has a fundamental frequency of 5100 Hz. If the speed of sound in air is 340 ms^{-1}, the length of the whistle, in cm, is

(A) 5/3 (B) 10/3 (C) 5 (D) 20/3

Solution : (B)

25. To determine the coefficient of friction between a rough surface and a block, the surface is kept inclined at 45° and the block is released from rest. The block takes a time t in moving a distance d. The rough surface is then replaced by a smooth surface and the same experiment is repeated. The block now takes a time t/2 in moving down the same distance d. The coefficient of friction is

(A) 3/4 (B) 5/4 (C) 1/2 (D) $1/\sqrt{2}$

Solution : (A)

26. A smooth massless string passes over a smooth fixed pulley. Two masses m_1 and m_2, ($m_1 > m_2$) are tied at the two ends of the string. The masses are allowed to move under gravity starting from rest. The total external force acting on the two masses is

(A) $(m_1 + m_2)g$ (B) $\frac{(m_1 - m_2)^2}{m_1 + m_2}g$ (C) $(m_1 - m_2)g$ (D) $\frac{(m_1 + m_2)^2}{m_1 - m_2}g$

Solution : (C)

27. A wooden block is floating on water kept in a beaker. 40% of the block is above the water surface. Now the beaker is kept inside a lift that starts going upward with acceleration equal to g/2. The block will then

(A) sink (B) float with 10% above the water surface
(C) float with 40% above the water surface (D) float with 70% above the water surface

Solution : (B)

28. In which of the following phenomena, the heat waves travel along straight lines with the speed of light ?

(A) thermal conduction (B) forced convection (C) natural convection (D) thermal radiation

Solution : (D)

29. An artificial satellite moves in a circular orbit around the earth. Total energy of the satellite is given by E. The potential energy of the satellite is

(A) – 2E (B) 2E (C) 2E/3 (D) –2E/3

Solution : (B)

30. A proton of mass m and charge q is moving in a plane with kinetic energy E. If there exists a uniform magnetic field B, perpendicular to the plane of the motion, the proton will move in a circular path of radius

(A) $\frac{2Em}{qB}$ (B) $\frac{\sqrt{2Em}}{qB}$ (C) $\frac{\sqrt{Em}}{2qB}$ (D) $\sqrt{\frac{2Eq}{mB}}$

Solution : (B)

31. A particle moves with constant acceleration along a straight line starting from rest. The percentage increase in its displacement during the 4^{th} second compared to that in the 3^{rd} second is

(A) 33% (B) 40% (C) 66% (D) 77%

Solution : (B)

32. A car is moving with a speed of 72 km-hour^{-1} towards a roadside source that emits sound at a frequency of 850 Hz. The car driver listens to the sound while approaching the source and again while moving away from the source after crossing it. If the velocity of sound is 340 ms^{-1}, the difference of the two frequencies, the driver hears is

(A) 50 Hz (B) 85 Hz (C) 100 Hz (D) 150 Hz

Solution : (C)

33. For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur ?

(A) isobar of original nucleus is produced

(B) isotope of the original nucleus is produced

(C) nuclei with higher atomic number than that of the original nucleus is produced

(D) nuclei with lower atomic number than that of the original nucleus is produced

Solution : (B)

34. Same quantity of ice is filled in each of the two metal containers P and Q having the same size, shape and wall thickness but made of different materials. The containers are kept in identical surroundings. The ice in P melts comlpletely in time t_1 whereas that in Q takes a time t_2. The ratio of thermal conductivities of the materials of P and Q is

(A) $t_2 : t_1$ (B) $t_1 : t_2$ (C) $t_1^2 : t_2^2$ (D) $t_2^2 : t_1^2$

Solution : (A)

35. When a particle executing SHM oscillates with a frequency ν, then the kinetic energy of the particle

(A) changes periodically with a frequency of ν

(B) changes periodically with a frequency of 2ν

(C) changes periodically with a frequency of $\nu/2$

(D) remains constant

Solution : (B)

36. A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now moved farther apart by pulling at them by means of insulating handles then

(A) the energy stored in the capacitor decreases

(B) the capacitance of the capacitor increases

(C) the charge on the capacitor decreases

(D) the voltage across the capacitor increases

Solution : (D)

37. The ionization energy of hydrogen is 13.6 eV. The energy of the photon released when an electron jumps from the first excited state (n = 2) to the ground state of a hydrogen atom is

(A) 3.4 eV (B) 4.53 eV (C) 10.2 eV (D) 13.6 eV

Solution : (C)

38. A uniform rod is suspended horizontally from its mid-poind. A piece of metal whose weight is W is suspended at a distance l from the mid-point. Another weight W_1 is suspended on the other side at a distance l_1 from the mid-point to bring the rod to a horizontal position. When W is completely immersed in water, W_1 needs to be kept at a distance l_2 from the mid-point to get the rod back into horizontal position. The specific gravity of the metal piece is

(A) $\frac{W}{W_1}$ (B) $\frac{Wl_1}{Wl - W_1 l_2}$ (C) $\frac{l_1}{l_1 - l_2}$ (D) $\frac{l_1}{l_2}$

Solution : (C)

39. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be

(A) $2r\cos\left(\frac{\theta}{2}\right)$ (B) $2r\cot\left(\frac{\theta}{2}\right)$ (C) $2r\tan\left(\frac{\theta}{2}\right)$ (D) $2r\sin\left(\frac{\theta}{2}\right)$

Solution : (D)

40. A metal rod is fixed rigidly at two ends so as to prevent its thermal expansion. If L, α and Y respectively denote the length of the rod, coefficient of linear thermal expansion and Young's modulus of its material, then for an increase in temperature of the rod by ΔT, the longitudinal stress developed in the rod is

(A) inversely proportional to α
(B) inversely proportional to Y
(C) directly proportional to $\frac{\Delta T}{Y}$
(D) independent of L

Solution : (D)

41. The intermediate image formed by the objective of a compound microseope is

(A) real, inverted and magnified
(B) real, erect and magnified
(C) virtual, erect and magnified
(D) virtual, inverted and magnified

Solution : (A)

42. Two coherent monochromatic beams of intensities I and 4I respectively are superposed. The maximum and minimum intensities in the resulting pattern are

(A) 5I and 3I
(B) 9I and 3I
(C) 4I and I
(D) 9I and I

Solution : (D)

43. The energy of gamma (γ) ray photon is E_γ and that of an X-ray photon is E_x. If the visible light photon has an energy of E_v, then we can say that

(A) $E_x > E_\gamma > E_v$
(B) $E_\gamma > E_v > E_x$
(C) $E_\gamma > E_x > E_v$
(D) $E_x > E_v > E_\gamma$

Solution : (C)

44. A uniform solid spherical ball is rolling down a smooth inclined plane from a height h. The velocity attained by the ball when it reaches the bottom of the inclined plane is v. If the ball is now thrown vertically upwards with the same velocity v, the maximum height to which the ball will rise is

(A) 5h/8
(B) 3h/5
(C) 5h/7
(D) 7h/9

Solution : (C)

45. If the bandgap between valence band and conduction band in a material is 5.0 eV, then the material is

(A) semiconductor
(B) good conductor
(C) superconductor
(D) insulator

Solution : (D)

Category : II

46. An object is palced 30 cm away from a convex lens of focal length 10 cm and a sharp image is formed on a screen. Now a concave lens is placed in contact with the convex lens. The screen now has to be moved by 45cm to get a sharp image again. The magnitude of focal length of the concave lens is (in cm)

(A) 72
(B) 60
(C) 36
(D) 20

Solution : (D)

47. A 10 Watt electric heater is used to heat a container filled with 0.5 kg of water. It is found that the temperature of water and the container rises by 3°K in 15 minutes. The container is then emptied, dried and filled with 2 kg of oil. The same heater now raises the temperature of container-oil system by 2° K in 20 kinutes. Assuming that there is no heat loss in the process and the specific heat of water as 4200 $Jkg^{-1} K^{-1}$, the specific heat of oil in the same unit is equal to

(A) 1.50×10^3
(B) 2.55×10^3
(C) 3.00×10^3
(D) 5.10×10^3

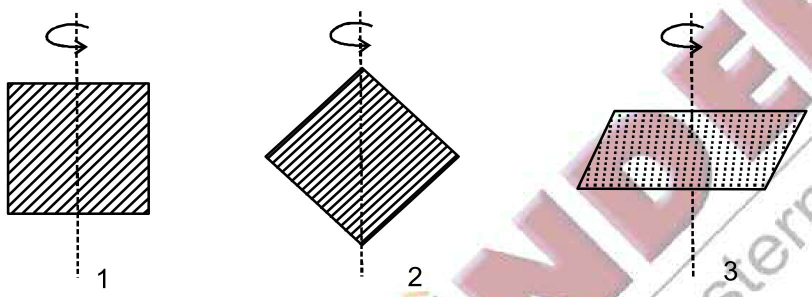
Solution : (B)

48. A solid uniform sphere resting on a rough horizontal plane is given a horizontal impulse directed through its center so that it starts sliding with an initial velocity v_0. When it finally starts rolling without slipping the speed of its center is

(A) $\frac{2}{7}v_0$ (B) $\frac{3}{7}v_0$ (C) $\frac{5}{7}v_0$ (D) $\frac{6}{7}v_0$

Solution : (C)

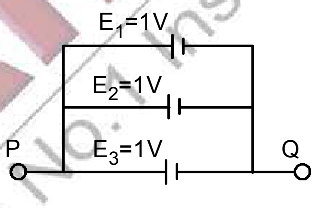
49. Three identical square plates rotate about the axes shown in the figure in such a way that their kinetic energies are equal. Each of the rotation axes passes through the centre of the square. Then the ratio of angular speed $\omega_1 : \omega_2 : \omega_3$ is



(A) 1 : 1 : 1 (B) $\sqrt{2}:\sqrt{2}:1$ (C) $1:\sqrt{2}:1$ (D) $1:2:\sqrt{2}$

Solution : (B)

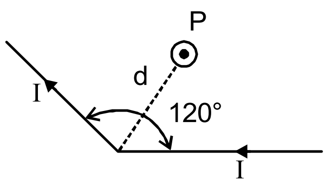
50. A circuit consists of three batteries of emf E_1 = 1V, E_2 = 2V and E_3 = 3V and initernal resistances 1Ω, 2Ω and 1Ω respectively which are connected in parallel as shown in the figure. The potential difference between points P and Q is



(A) 1.0 V (B) 2.0 V (C) 2.2 V (D) 3.0 V

Solution : (B)

51. A long conducting wire carrying a current I is bent at 120° (see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is (μ_0 is the permeability of free space)



(A) $\frac{3\mu_0 I}{2\pi d}$ (B) $\frac{\mu_0 I}{2\pi d}$ (C) $\frac{\mu_0 I}{\sqrt{3}\pi d}$ (D) $\frac{\sqrt{3}\mu_0 I}{2\pi d}$

Solution : (D)

52. To determine the composition of a bimetallic alloy, a sample is first weighed in air and then in water. These weights are found to be W_1 and W_2 respectively. If the densities of the two constituent metals are ρ_1 and ρ_2 respectively, then the weight of the first metal in the sample is (where ρ_w is the density of water)

(A) $\frac{\rho_1}{\rho_w(\rho_2-\rho_1)}\left[W_1(\rho_2-\rho_w)-W_2\rho_2\right]$ (B) $\frac{\rho_1}{\rho_w(\rho_2+\rho_1)}\left[W_1(\rho_2-\rho_w)+W_2\rho_2\right]$

(C) $\frac{\rho_1}{\rho_w(\rho_2-\rho_1)}\left[W_1(\rho_2+\rho_w)-W_2\rho_1\right]$ (D) $\frac{\rho_1}{\rho_w(\rho_2-\rho_1)}\left[W_1(\rho_1-\rho_w)-W_2\rho_1\right]$

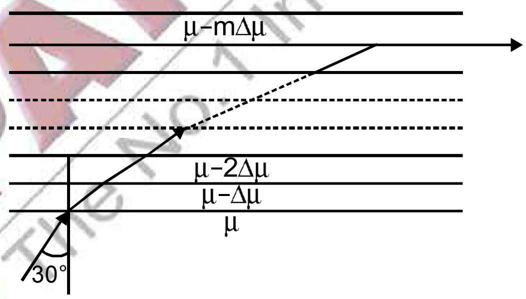
Solution : (A)

53. Sound waves are passing through two routes-one in straight path and the other along a semicircular path of radius r, and are again combined into one pipe and superposed as shown in the figure. If the velocity of sound waves in the pipe is v, then frequencies of resultant waves of maximum amplitude will be integral multiples of

(A) $\frac{v}{r(\pi-2)}$ (B) $\frac{v}{r(\pi-1)}$ (C) $\frac{2v}{r(\pi-1)}$ (D) $\frac{v}{r(\pi+1)}$

Solution : (A)

54. A glass slab consists of thin uniform layers of progressively decreasing refractive indices RI (see figure)such that the RI of any layer is μ-$m\Delta\mu$. Here μ and $\Delta\mu$ denote the RI of 0^{th} layer and the difference in RI between any two consecutive layers,respectively. The integer m = 0, 1, 2, 3...denotes the number of the successive layers. A ray of light from the 0^{th} layer enters the 1^{th} layer at an angle of incidence of 30°. After undergoing the m^{th} refraction, the ray emerges parallel to the interface. If μ = 1.5 and $\Delta\mu$ = 0.015, the value of m is



(A) 20 (B) 30 (C) 40 (D) 50

Solution : (D)

55. The de Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass is 0.5 MeV)

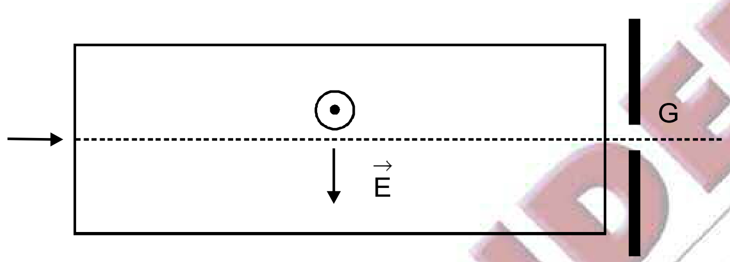
(A) 1 : 50 (B) 1 : 20 (C) 20 : 1 (D) 50 : 1

Solution : (C)

Category : III

Q.56 to Q.60 carry two marks each, for which one or more than one options may be correct. Marking of correct options will lead to a maximum mark of two on pro rata basis. There will be no negative marking for these questions. However, any marking of wrong option will lead to award of zero mark against the respective question – irrespective of the number of correct options marked.

56. A stream of electrons and protons are directed towards a narrow slit in a screen (see figure). The intervening region has a uniform field $\vec{E}$ (vertically downwards) and a uniform magnetic field $\vec{B}$ (out of the plane of the figure) as shown . Then



(A) electrons and protons with speed $\frac{|\vec{E}|}{|\vec{B}|}$ will pass through slit.

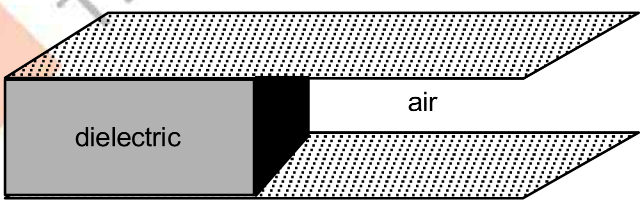
(B) protons with speed $\frac{|\vec{E}|}{|\vec{B}|}$ will pass through the slit, electrons of the same speed will not.

(C) neither electrons nor protons will go through the slit irrespective of their speed.

(D) electrons will always be deflected upwards irrespective of their speed.

Solution : (C,D)

57. Half of the space between the plates of a parallel-plate capacitor is filled with a dielectric material of dielectric constant K. The remaining half contains air as shown in the figure.



(A) electric field in the dielectric-filled region is higher than that in the air-filled region

(B) on the two halves of the bottom plate the charge densities are unequal

(C) charge on the half of the top plate above the air-filled part is $\frac{Q}{k+1}$

(D) capacitance of the capacitor shown above is $(1 + K)\frac{C_0}{2}$, where C_0 is the capacitance of the same capacitor with the dielectric removed.

Solution : (B,C,D)

58. Find the correct statement(s) about photoelectric effect.

(A) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons

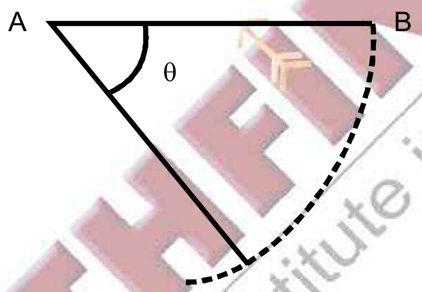
(B) Einstein analysis gives a threshold frequency above which no electron can be emitted

(C) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation

(D) The maximum kinetic energy of electrons does not depend on the intensity of radiation

Solution : (A,D)

59. A thin rod AB is held horizontally so that it can freely rotate in a vertical plane about the end A as shown in the figure. The potential energy of the rod when it hangs vertically is taken to be zero. The end B of the rod is released from rest from a horizontal position. At the instant the rod makes an angle θ with the horizontal,



(A) the speed of end B is proportional to $\sqrt{\sin\theta}$

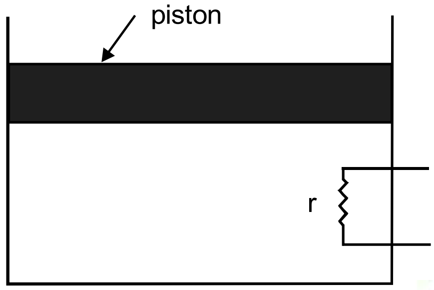
(B) the potential energy is proportional to $(1 - \cos\theta)$

(C) the angular acceleration is proportional to $\cos\theta$

(D) the torque about A remains the same as its initial value

Solution : (A,C)

60. A heating element of resistance r is fitted inside an adiabatic cylinder which carries a frictionless piston of mass m and cross-section A as shown in diagram.The cylinder contains one mole of an ideal diatomic gas. The current flows through the element such that the temperature rises with time t as $\Delta T = \alpha t + \frac{1}{2}\beta t^2$ (α and β are constants), while pressure remains constant. The atmospheric pressure above the piston is P_0. Then



(A) the rate of increase in internal energy is $\frac{5}{2}R(\alpha + \beta t)$

(B) the current flowing in the element is $\sqrt{\frac{5}{2r}R(\alpha + \beta t)}$

(C) the piston moves upwards with constant acceleration

(D) the piston moves upwards with constant speed

Solution : (A,C)